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DYNAMIC MODELING OF
POST FAILURE CONDITIONS
OF REINFORCED CONCRETE SUBJECTED TO BLAST

MICHAEL F. LEONDI

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
LARGE CALIBER
WEAPON SYSTEMS LABORATORY
DOVER, NEW JERSEY

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A dynamic modeling analysis of post-failure fragments of reinforced concrete subjected to blast overpressures was performed. The analysis related the physical (constitutive) properties as well as the geometric relationships of the model and the prototype. Results indicated that a replica model can provide rational test data if all the mechanical and geometric properties of the reinforced concrete element are reflected in the model.

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INTRODUCTION

In support of an Army-wide modernization program, the Army Armament Research and Development Command, Dover, NJ, is engaged in the development of safety criteria as a project entitled, "Safety Engineering in Support of Ammunition Plants."

Prior to investigating the explosive fragmentation of reinforced concrete structures (one aspect of the safety engineering program), a feasibility study of sub-scale dynamic modeling was conducted to determine:

      1.   The applicability of sub-scale modeling to random mass fragments of reinforced concrete subjected to excessive blast loads.

      2.   The physical relationships between the model and the prototype in constitutive properties, panel geometry, and boundary conditions as well as the resulting fragment weight/shape, density, and distribution.

PROBLEM DEFINITION AND SOLUTION

In munitions manufacturing facilities, reinforced concrete plays an essential role as a building material for both dividing walls (between quantities of explosive) and exterior walls and roofs.

Mechanics of Fragmentation (ref. 1)

When a reinforced concrete element is substantially overloaded by a blast wave, the element fails and concrete fragments (post-failure) are displaced at high velocities (fig. 1).

Failure of an element is characterized by the dispersal of concrete fragments formed by the cracking and displacement of the concrete between the donor and receiver layers of the reinforcement. As an element deflects and the concrete begins to crush, the compression stresses normally resisted by the concrete are transferred to the reinforcement. With increased deflections, these compression forces tend to buckle the reinforcement outward, initiating rapid disintegration of the element.

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The velocity of individual fragments varies and depends on: (1) the magnitude of the excess (or blast) impulse minus the flexural impulse capacity of the element (area under the resistance-time curve), (2) the mass of the fragment, (3) the location of the fragment prior to collapse, (4) the interaction between fragments during their flight, and (5) the strength and time history of the compressive stress wave transmitted through the wall as the blast wave is reflected. Although the velocities of individual fragments differ, the average translational velocity, v_s (avg), of the debris after complete failure can be approximated from the excess impulse, i_e, and the unit mass, m, of the barrier; i.e., the momentum of the wall after collapse is numerically equal to the excess impulse, $i_e = mv_f$ (avg).

Also,

$$i^2 = C_u \frac{(pHd_c^3 f_{ds}) + C_f d_c^2 v_f^2}{H}$$

where

    i = applied unit blast impulse

    pH = reinforcement ratio in the horizontal direction

    d_c = distance between the centroids of the compression and tension reinforcement.

    f_{ds} = dynamic design stress for the reinforcement

    H = span height

    v_f = maximum velocity of the post-failure fragments

    C_u = impulse coefficient for ultimate deflection X_u

    C_f = post-failure fragment coefficient

The reflected shock wave is also transmitted through the wall as a compressive wave, and reflects from the rear surface as a tension wave which partially cancels the incoming compressive wave. If the shock strength is great enough and its decay rate rapid enough, a net tensile stress will occur at some distance from the rear surface. The concrete (which is weak in tension) will fail and spall off. The thickness of the spall and its velocity can be estimated

from knowledge of the reflected wave properties and the constitutive properties of the concrete.

Dynamic Modeling of Reinforced Concrete Elements

The following exercise defines the physical similarity between reinforced concrete elements in the model and the prototype subjected to blast overloading as described in reference 2. The constitutive, geometric, and mechanical parameters dictated by the scope of the problem are given in table 1.

PI TERMS FROM DIMENSIONAL HOMOGENEITY

By inspection (already dimensionless)

$$\pi^{13} = \psi$$

$$\pi^{14} = K$$

Technique from reference 2: <u>Matrix Solution</u>

$$F^0 L^0 T^0 \overset{d}{=} W^{a_1} R^{a_2} \rho_c^{a_3} \rho_s^{a_4} C_c^{a_5} T_c^{a_6} U_s^{a_7} T_s^{a_8} L_c^{a_9} L_s^{a_{10}} L_A^{a_{11}} E_s^{a_{12}}$$

$$E_c^{a_{13}} W_F^{a_{14}} V^{a_{15}}$$

Using matrix algebra (identity submatrix)

<u>Form a matrix:</u>

	W	R	ρ_c	ρ_s	C_c	T_c	U_s	T_s	L_c	L_s	L_A	E_s	E_c	W_F	V
	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
F	+1	0	+1	+1	+1	+1	+1	+1	0	0	0	+1	+1	+1	0
L	+1	+1	-4	-4	-2	-2	-2	-2	+1	+1	+1	-2	-2	+1	3
T	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0

<u>Matrix algebra steps:</u>

Step 1 - Divide row 3 by row 2.

Step 2 - Subtract new row 3 from row 1, and write new row 1.

Step 3 - Subtract row 1 from row 2.

Step 4 - Multiply row 3 by row 4 and add to row 2. Write new row 2.

Therefore, identity submatrix in the following matrix:

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
F	+1	0	0	0	+1	+1	+1	+1	0	0	0	+1	+1	+1	0
L	0	+1	0	0	-3	-3	-3	-3	+1	+1	+1	-3	-3	0	+3
T	0	0	+1	+1	0	0	0	0	0	0	0	0	0	0	0

Then, by inspection

$$a_1 = -a_5 - a_6 - a_7 - a_8 - a_{12} - a_{13} - a_{14}$$

$$a_2 = 3a_5 + 3a_6 + 3a_7 + 3a_8 - a_9 - a_{10} - a_{11} + 3a_{12} + 3a_{13} - 3a_{15}$$

$$a_3 = -a_4$$

Substituting in the original equation

$$F^0 L^0 T^0 \overset{d}{=} W^{-a_5 - a_6 - a_7 - a_8 - a_{12} - a_{13} - a_{14}} R^{3a_5 + 3a_6 + 3a_7 + 3a_8 - a_9 - a_{10} - a_{11} + 3a_{12} + 3a_{13} - 3a_{15}}$$

$$x \ \rho_c^{-a_4} \rho_s^{a_4} C_c^{a_5} T_c^{a_6} U_s^{a_7} T_s^{a_8} L_c^{a_9} L_s^{a_{10}} L_A^{a_{11}} E_s^{a_{12}}$$

$$x \ E_c^{a_{13}} W_F^{a_{14}} V^{a_{15}}$$

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Collecting terms of like exponent, we have the model law. This can be expressed as:

$$\left.\begin{array}{c} \dfrac{(W_F}{W} \\[4pt] \dfrac{V}{R^3} \\[4pt] \dfrac{\psi}{K} \end{array}\right\} \; f_i \quad \dfrac{\rho_s}{\rho_c,} \quad \dfrac{R^3 C_c}{W} \quad \dfrac{R^3 T_c}{W}, \quad \dfrac{R^3 U_s}{W}, \quad \dfrac{R^3 T_s}{W}, \quad \dfrac{L_c}{R}, \quad \dfrac{L_s}{R}, \quad \dfrac{L_A}{R}, \quad \dfrac{R^3 E_s}{W}, \quad \dfrac{R^3 E_c}{W}$$

The law can also be stated in the form of table 2.

SCALE FACTORS

Various relations between scale factors can be determined from the dimensionless terms.

$\pi_1 \rightarrow \lambda_{\rho_s} = \lambda_{\rho_c} = 1$ (set equal to 1 for ease in fabricating model)

$\pi_2, \; \pi_3, \; \pi_4, \; \pi_5, \; \pi_6, \; \pi_7 \rightarrow \dfrac{\lambda}{\lambda^3 R} = \lambda_{C_c} = \lambda_{T_c} = \lambda_{U_s} = \lambda_{T_s}$

$$= \lambda_{E_s} = \lambda_{E_c}$$

$\pi_8, \; \pi_9, \; \pi_{10} \rightarrow \lambda_{L_c} = \lambda_{L_s} = \lambda_{L_A} = \lambda_R$

$\pi_{11} \rightarrow \lambda_{W_F} = \lambda_W$

$\pi_{12} \rightarrow \lambda_V = \lambda^3 R$

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These interrelations are all satisified by a replica model[1] with geometry identical to the prototype. Although wave transmission effects are not shown in scale, they will scale properly because all constitutive properties and densities are unchanged.

CONCLUSIONS AND RECOMMENDATIONS

The preceding similarity exercise has demonstrated that

1. A replica model accurately reflects the performance of a prototype system.

2. Failure modes of a replica model are representative for geometrically similar panels.

3. The model also exhibits similar relationships for fragment mass and velocity, distribution, and debris density.

For economic reasons and ease in testing, it is recommended that subscale replica models be used with a scale factor as small as possible without sacrificing the quality of the model.

REFERENCES

1. "Structures to Resist the Effects of Accidental Explosions," Dept of the Army TM 5-1300, June 2, 1969.

2. W. E. Baker, P. S. Westing, F. T. Dodge, Similarity Methods in Engineering Dynamics, SPARTAN BOOKS, San Antonio, TX, 1973.

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[1]A replica model is a physical model of a prototype which is geometrically similar in all respects and employs the same materials in the same locations as the prototype (ref. 2).

Table 1. List of parameters for explosive fragmentation of reinforced concrete elements

No.	Parameter	Dimension	Description
1	W	FL	Energy in blast source
2	R	L	Radial distance of blast
3	ρ_c	FT^2/L^4	Density concrete
4	ρ_s	FT^2/L^4	Density steel
5	C_c	F/L^2	Compressive strength (concrete)
6	T_c	F/L^2	Tensile strength (concrete)
7	U_s	F/L^2	Ultimate strength (steel)
8	T_s	F/L^2	Tensile strength (steel)
9	L_c	L	Characteristic geom. of element
10	L_s	L	Characteristic geom. of rebar
11	L_A	L	Aggregate size
12	E_s	F/L^2	Elastic moduli (steel)
13	E_c	F/L^2	Elastic moduli (concrete)
14	W_f	FL	Energy imparted into fragments
15	V	L^3	Vol of fragments
16	ψ	--	Distribution function of fragments
17	K	--	% energy absorbed by element

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Table 2. PI terms - explosive fragmentation

$\pi_1 = \rho_s/\rho_c$

$\pi_2 = R^3 Cc/W$

$\pi_3 = R^3 Tc/W$

$\pi_4 = R^3 Us/W$ — CONSTITUTIVE SIMILARITY

$\pi_5 = R^3 Ts/W$

$\pi_6 = R^3 Es/W$

$\pi_7 = R^3 Ec/W$

$\pi_8 = Lc/R$

$\pi_9 = Ls/R$ — GEOMETRIC SIMILARITY

$\pi_{10} = LA/R$

$\pi_{11} = W_F/W$ — SIMILAR ENERGY TRANSFERS

$\pi_{12} = V/R^3$ — SIMILAR MASS TRANSFERS

$\pi_{13} = \psi$ — DISTRIBUTION FUNCTION

$\pi_{14} = K$ — % ENERGY ABSORBED

DONOR
(Explosive Charge)

REINFORCED CONCRETE
PROTECTIVE ELEMENT

R

LETHAL ZONE

PREDETONATION
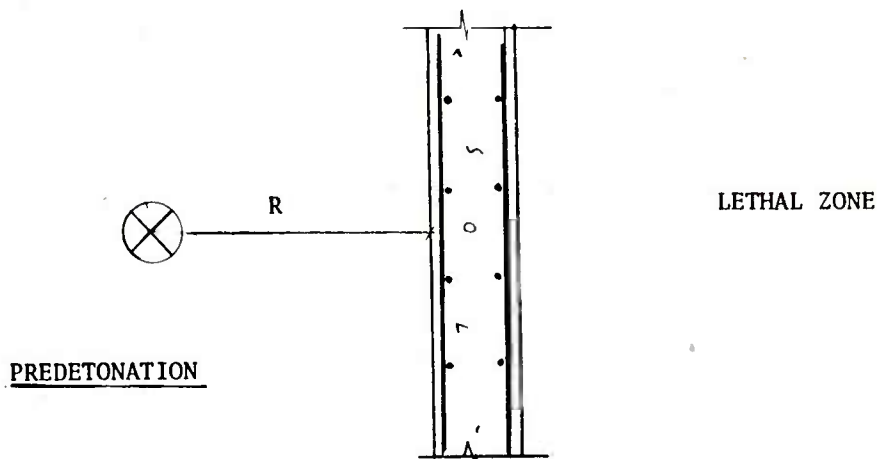
ENERGY AND IMPULSE BALANCES

Blast Energy In = Strain Energy + Sum of Fragment Kinetic Energies
= $\dfrac{\text{Sum of Fragment Kinetic Energies}}{(1 - K)(\text{where } K = \% \text{ Blast Energy Absorbed})}$

Applied Impulse = Flexural Impulse Capacity + Excess Impulse

ENERGY IN

BLAST
LOADING
P(t)

FRAGMENT ENERGY

$M_3 V_3$

$M_2 V_2$

$M_1 V_1$

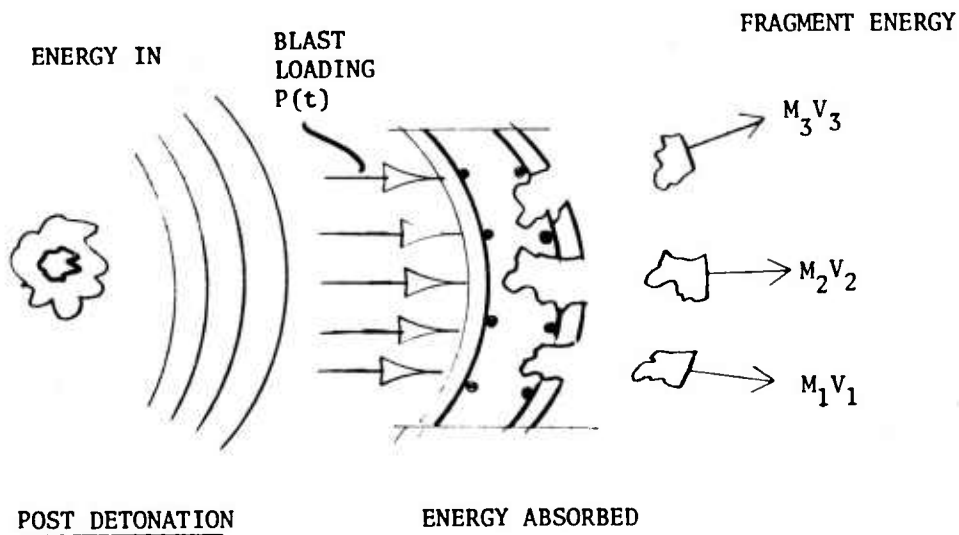POST DETONATION

ENERGY ABSORBED

Figure 1. Explosive fragmentation.

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